HEAT TRANSFER AND FLUID FRICTION IN BUNDLES OF
TVISTED TUBES
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The results of heat-transfer and friction studies in bundles of twisted tubes and rods with spiral wire-wrap spacers are analyzed, and recommendations are given for calculating the heat-transfer coefficient in heat exchangers using twisted tubes.

Heat-exchange equipment using twisted oval tubes has the optimum size and weight characteristics [1] owing to the intensification of heat transfer by swirling of the flow in the intertubular space of the equipment and by the flow inside the tubes. Heat transfer in bundles of twisted tubes has been investigated in several papers [1-3]. The generalization of the experimental heat-transfer data has resulted in the derivation of similarity equations with the use of various experimentally substantiated theoretical considerations. For example, the following criterion characterizing the ratio between the inertial and centrifugal forces in a bundle of twisted tubes has been proposed [4] on the basis of similarity theory and dimensional analysis on the assumption that the flow of the heat-transfer agent is swirled by the helical channels of the tubes according to the rigid-body law $u_{\mathrm{q}} / \mathrm{r}=$ const:

$$
\begin{equation*}
\mathrm{Fr}=S^{2} / 2 \pi^{2} d d_{\mathrm{e}} \tag{1}
\end{equation*}
$$

which can be written as follows to within a constant:

$$
\begin{equation*}
\mathrm{Fr}_{\mathrm{M}}=S^{2} / d d_{\mathrm{e}} . \tag{2}
\end{equation*}
$$

The complex geometrical characteristic (2) of the bundle has made it possible to generalize the experimental data on heat transfer and fluid friction to geometrically nonsimilar equipment as well [2]. We have used the number $\mathrm{Fr}_{\mathrm{M}}$ to develop a method for the calculation of heat transfer on the basis of the introduction of an effective wall-layer thickness

$$
\begin{equation*}
\delta=0.5\left(1+3.6 \mathrm{Fr}^{-0.357}\right)^{-4} d_{\mathrm{e}} \tag{3}
\end{equation*}
$$

as the governing length in the flow of the heat-transfer agent in a bundle of twisted tubes. In this case the experimental heat-transfer data for the bundles at numbers $\operatorname{Re}_{\delta}>5 \cdot 10^{2}$ and $\mathrm{Fr}_{\mathrm{M}}>90$ are generated by the relation for circular tubes [2]

$$
\begin{equation*}
\mathrm{Nu}_{\delta}=0.02 \operatorname{Re}_{\delta}^{0.8} \operatorname{Pr}^{0.4}, \tag{4}
\end{equation*}
$$

which is plotted in Fig. 1. In the transition flow region $\operatorname{Re}_{\delta}<5 \cdot 10^{2}, \mathrm{Fr}_{\mathrm{M}} \geq 64$ the heat transfer is given by the expression

$$
\begin{equation*}
\mathrm{Nu}_{\delta}=6.47 \mathrm{Fr}_{\mathrm{M}}^{-0.845} \operatorname{Re}_{\delta}^{n} \operatorname{Pr}^{0.4} \tag{5}
\end{equation*}
$$

where

$$
\begin{equation*}
n=0.212 \mathrm{Fr}_{\mathrm{M}}^{0.194} \tag{6}
\end{equation*}
$$

and for $\mathrm{Fr}_{\mathrm{M}} \geq 924$ we have $\mathrm{n}=0.8$.
As the number $\mathrm{Fr}_{\mathrm{M}}$ is decreased, the power exponent of $\mathrm{Re}_{\delta}$ in (5) decreases from 0.8 at $\mathrm{Fr}_{\mathrm{M}}=924$ to 0.475 at $\mathrm{Fr}_{\mathrm{M}}=64$. For $\mathrm{Fr}_{\mathrm{M}}<90$, according to the data of $[1,2]$, the heat

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Fig. 1. Comparison of experimental heat-transfer data for bundles with different numbers of tubes, using a flow model with the concept of the characteristic wall-layer thickness. 1) Eq. (4), $\mathrm{N} \geq 37$; 2) Eq. (5), $\mathrm{Fr}_{\mathrm{M}}=232$; 3, 4) Eqs. (7) and (5) for $\mathrm{Fr}_{\mathrm{M}}=64$; 5) Eq. (20), $N=19 ; 6-8)$ endpoints of the experimental range of numbers $\operatorname{Re}=$ $5 \cdot 10^{3}-6 \cdot 10^{4}$ at $\mathrm{Fr}_{\mathrm{M}}=26,98$, 235 for bundles with $\mathrm{N}=19$.

$$
\begin{aligned}
& \mathrm{Nu} / \mathrm{Nu} \mathrm{tu} \\
& 2,4 \\
& \hline
\end{aligned}
$$

Fig. 2. Comparison of experimental heat-transfer data at Re $=10^{4}$ for bundles with different numbers of tubes, using a flow model with allowance for the transverse component of the velocity. 1-3) Eqs. (23), (25) (26); 4,5) experimentaldata [1-3] for bundles with $N \geq 37$ and $N=19$, respectively; 6, 7) data of [6] for the central rod and peripheral rods with spiral wire-wrap spacers, $N=7$.


Fig. 3. The same as Fig. 2, for $\operatorname{Re}=5 \cdot 10^{3}$ and $10^{5}$. 1) Eq. (23); 2, 3) Eqs. (25) and (26), $\operatorname{Re}=10^{5}$; 4, 5) the same, $\operatorname{Re}=5 \cdot 10^{3}$; 6, 7) data of [6] for the central and peripheral rods with spiral wire-wrap spacers, $\left.\mathrm{N}=7, \operatorname{Re}=10^{5} ; 8,9\right)$ the same, $\operatorname{Re}=5 \cdot 10^{3}$.
transfer in the bundle is further intensified (Fig. 1) and is described by the following equation for $\mathrm{Fr}_{\mathrm{M}}=64, \mathrm{Re}_{\delta}>500$ :

$$
\begin{equation*}
\mathrm{Nu}_{\delta}=0.0248 \operatorname{Re}_{8}^{0,8} \operatorname{Pr}^{0,4} \tag{7}
\end{equation*}
$$

In Eqs. (4), (5), and (7) $\mathrm{Nu}_{\delta}=\alpha \delta / \lambda, \operatorname{Re}_{\delta}=\rho u_{\mathrm{av}^{\delta}} \delta / \mu$.


Fig. 4. Influence of the number $\mathrm{Fr}_{\mathrm{M}}$ on the relative friction factor at $\operatorname{Re}=10^{4}$. 1, 2) Functions (23) and (33); 3) function $\xi$ (14), referred to $\xi_{f r}$; experimental data: 4) $N \geq 37$ [1, 2]; 5) $\mathrm{N}=19[10,11]$; 6) $\mathrm{N}=127[10]$; 7) $\mathrm{N}=19[3]$; 8) $\mathrm{N}=7$ $[6] ; 9) \mathrm{N}=127$ [12]; 10) the same [13].

Our processing of the experimental data, which expands the heat-transfer modeling possibilities, is based on the results of an experimental study of the flow structure in bundles of twisted tubes. It has been shown [4] that when a heat-transfer medium (coolant) flows in bundles of twisted tubes, a thin wall layer is formed, and the flow core has roughly a constant velocity. The wall-layer thickness decreases with a decrease in $\mathrm{Fr}_{\mathrm{M}}$. In this case the flow in the wall layer swirls according to the law $u_{\tau} / r=$ const, and the swirl in the flow core is determined by the interaction of the helical flows of the adjacent tubes [5]. These distinctive features of the flow also confirm the flow model postulated in [3], which is based on semiempirical theories of Prandtl turbulence and treats the interaction of two flows directed at an angle relative to one another in an equivalent plane channel. In the middle zone of this channel, where the tangential velocity vector changes direction, turbulence and secondary streaming are generated in addition to the typical mechanism of the onset of turbulence for a straight channel. Here the tangential stresses decompose into an axial component and a transverse-tangential component [3]:

$$
\begin{equation*}
\tau_{x}=\left(\mu+\mu_{\mathrm{T} x}\right)\left(d u_{x} / d y\right), \tau_{z}==\left(\mu+\mu_{\mathrm{T} z}\right)\left(d u_{z} / d y\right) . \tag{8}
\end{equation*}
$$

Setting $\mu_{T x}=\mu_{T z}=\mu_{T}$ and invoking the hypothesis of $Y u$. A. Koshmarov for the case of the rotation-translation (swirling) motion of a fluid between two coaxial cylinders spinning one relative to the other, we find

$$
\begin{equation*}
\mu_{\mathrm{r}}=\rho l^{2}\left(\left|d u_{x} / d y\right|+\left|d u_{z} / d y\right|\right) \tag{9}
\end{equation*}
$$

We have $Z=x y$ near the wall and $Z=x y_{1}=$ const in the middle zone of the channel; $y_{1}$ corresponds to the point with $\mathrm{u}_{\mathrm{z}}=\mathrm{u}_{\text {zmax }}$.

The tangential stress at the wall can be decomposed into two components: one axial $\tau_{\mathrm{XW}}$ and one transverse-tangential $\tau_{\mathrm{zW}}$. In the center of the channel the tangential stress consists of the transverse-tangential component $\tau_{z 0}$ only. A steady stabilized flow of an incompressible fluid in a plane channel is known to have a linear height distribution of the tangential stress. Similarly, the transverse-tangential component $\tau_{z}$ of the tangential stress varies linearly from $\tau_{z w}$ at the wall to $\tau_{z 0}$ on the axis of the channel. In this case the transverse-tangential stress $\tau_{z}=0$ at the point $y=y_{1}$.

Thus, for stabilized flow we have

$$
\begin{equation*}
\tau_{x}=\tau_{\mathrm{xw}}\left(1-y / y_{0}\right), \tau_{z}=\tau_{\mathrm{zw}}+\left(\tau_{z 0}-\tau_{\mathrm{zw}}\right)\left(y / y_{0}\right), \tag{10}
\end{equation*}
$$

where $y_{o}$ is the distance from the channel axis. Solving the system of equations (10) with allowance for Eqs. (8) and (9), we can determine the fields of the axial and transversetangential components of the velocities for given numbers $\operatorname{Re}$ and $\Gamma=\bar{u}_{z} / \bar{u}_{x}=\pi^{2} / \frac{S}{d} \frac{S}{d e}=$ $\pi^{2} / \mathrm{Fr}_{\mathrm{M}}$, along with the total fluid-friction losses, from the force balance equation

$$
\begin{equation*}
\Delta p_{\Sigma}=\Delta p_{x}+\Delta p_{z}=\left[\tau_{\mathrm{xw}}+\left(\tau_{z w}+\tau_{z 0} K\right) \Gamma\right] \Delta x / y_{0}, \tag{11}
\end{equation*}
$$

where the coefficient $K$ characterizes the true geometry of the unit cell of the bundle.
In determining the relation for calculation of the heat transfer, we consider the energy equation for one-dimensional flow in the direction of the total velocity vector, the magnitude of which is given as $u_{\Sigma}=\sqrt{u_{x}^{2}+u_{z}^{2}}$ :

$$
\begin{equation*}
\frac{\partial}{\partial y}\left[\left(\lambda+\lambda_{\mathrm{T}}\right)\left(\frac{\partial T}{\partial y}\right)\right]=c_{p} \rho u_{\Sigma}\left(\frac{\partial T}{\partial x}\right) \tag{12}
\end{equation*}
$$

In dimensionless form, for $q_{W}=$ const we obtain

$$
\begin{equation*}
\mathrm{Nu}=4 / \int_{0}^{1} \frac{\left(\int_{0}^{\bar{\eta}} U_{\mathrm{\Sigma}} d \bar{\eta}\right)^{2}}{1+\frac{\operatorname{Pr}^{\mathrm{Pr}_{\mathrm{r}}}}{\mu_{\mathrm{r}}}} d \bar{\eta} \tag{13}
\end{equation*}
$$

where $U_{\Sigma}=u_{\Sigma} / \bar{u}_{\Sigma}$ and $\operatorname{Pr}_{T}$ is the turbulent Prandtl number. The integration in Eq. (13) is carried out by layers: 1) the viscous layer $\mu_{\mathrm{T}} / \mu<1$; 2) $\mu_{\mathrm{T}} / \mu>1,2=x y$; 3) $\mu_{\mathrm{T}} / \mu>1$, $Z=x y_{1}=$ const.

The results of calculations of Eqs. (11) and (13) are approximated by expressions that generalize the experimental data [3] for bundles of 19 tubes with $m=0.45$ and $\mathrm{Fr}_{\mathrm{M}}=26$, 98 , $235(\mathrm{~S} / \mathrm{d}=4.15,8.3,12.45)$ in the stabilized flow section:

$$
\begin{gather*}
\xi=\frac{0.25}{\operatorname{Re}^{0,22}}\left[\left(1+\frac{\pi^{2}}{0.9 \mathrm{Fr}_{\mathrm{m}}}\right)^{1,5}+\frac{100}{\operatorname{Fr}_{\mathrm{m}}^{1,25}}\right] \psi  \tag{14}\\
\mathrm{Nu}=0.035 \operatorname{Re}^{0.75}\left(1+\pi^{2} / 0.5 \mathrm{Fr}_{\mathrm{m}}\right)^{0.4}\left(1+1.3 / \mathrm{Fr}_{\mathrm{m}}^{0.6}\right)\left(T_{\mathrm{w}}\left(T_{\mathrm{f}}\right)^{-n},\right. \tag{15}
\end{gather*}
$$

where $\mathrm{Nu}=\alpha d_{\mathrm{e}} / \lambda ; \operatorname{Re}=\rho u_{\mathrm{av}} d_{\mathrm{e}} / \mu$.
In Eqs. (14) and (15), for $4.15<\mathrm{S} / \mathrm{d}<8.3$

$$
\begin{gather*}
\psi=1-\left(1-\psi_{0}\right)(S / d-4.15) / 4.15  \tag{16}\\
\psi_{0}=\left(T_{\mathrm{W}} / T_{\mathrm{f}}\right)^{-(\lg \operatorname{Re}-4)}, \tag{17}
\end{gather*}
$$

and for $\mathrm{S} / \mathrm{d} \geqslant 8.3$

$$
\begin{equation*}
\psi=\psi_{0} \tag{18}
\end{equation*}
$$

For $4.15<S / d<12.45$

$$
\begin{equation*}
n=0.55-0.0663(12.45-S / d) \tag{19}
\end{equation*}
$$

for $\mathrm{S} / \mathrm{d}>12.45$ we have $\mathrm{n}=0.55$, and for $\mathrm{S} / \mathrm{d}<4.15$ we have $\mathrm{n}=0$.
The experimental data for a 19 -tube bundle can be generalized by the equation

$$
\begin{equation*}
\mathrm{Nu}_{\delta}=0.0167 \operatorname{Re}_{\delta}^{0.8} \mathrm{Pr}^{0,4} \tag{20}
\end{equation*}
$$

the graph of which is $20 \%$ lower than the graph of Eq. (4) for bundles with $N \geqslant 37$ tubes (Fig. 1). This difference can be attributed to the influence of the peripheral row of tubes when different experimental and data-processing procedures are used. The heat-transfer conditions at the peripheral row of tubes of rods [6] are known to be analogous to the heattransfer conditions of fluid flow in an annular duct heated at one wall only, in which case the heat-transfer coefficient is reduced. This effect is evidently also observed in bundles of 19 twisted tubes when the heat-transfer coefficient is determined from the arithmetic-mean wall temperature of all 19 tubes and the calorimetric-mean temperature of the fluid. The heat-transfer coefficient in the bundles of $N \geqslant 37$ tubes is determined from measurements of the wall temperature of the central tube only.

The reduction of the heat-transfer coefficient at the peripheral row of rods with spiral wire-wrap spacers has also been confirmed in an investigation [6] of a seven-rod bundle with relative pitch $S / d=12,24,36\left(\mathrm{Fr}_{\mathrm{M}}=220,860\right.$, 1900), a relative tube spacing $\mathrm{P} / \mathrm{d}=1.237$, and $\operatorname{Pr}=0.7$. The heat-transfer coefficient was investigated separately for the central and peripheral rods in this work. The data can be compared with heat-transfer data for a bundle of twisted tubes, because the nature of the flow in rod bundles with spiral wire-wrap spacers and with twisted tubes is similar, and the heat- and mass-transfer characteristics are generalized by a common relation [7]. For the comparison we represent the experimental data for bundles of $N \geqslant 37$ tubes in the stabilized flow section by a similarity equation of the form [2]

$$
\begin{equation*}
\mathrm{Nu}=0.023 \operatorname{Re}^{0.8} \operatorname{Pr}^{0,4}\left(1+3.6 \mathrm{Fr}_{\mathrm{M}}^{-0,357}\right)\left(T_{\mathrm{W}} / T_{\mathrm{f}}\right)^{-0,55} \tag{21}
\end{equation*}
$$

We refer (21) to the relation for circular tubes

$$
\begin{equation*}
N u_{\mathrm{tu}}=0.023 \operatorname{Re}^{0,8} \operatorname{Pr}^{0,4}\left(T_{\mathrm{w}} / T_{\mathrm{f}}\right)^{-0,55} \tag{22}
\end{equation*}
$$

and arrive at the relation

$$
\begin{equation*}
A_{1}=\mathrm{Nu}_{/} \mathrm{Nu}_{\mathrm{tu}}=1+3.6 \operatorname{Fr}_{\mathrm{M}}^{-0,357} \tag{23}
\end{equation*}
$$

with which we compare the data of $[1,2]$ in Fig. 2. If we write Eq. (15) in the form

$$
\begin{equation*}
\mathrm{Nu}=0.044 \mathrm{Re}^{0,75}\left(1+\pi^{2} / 0.5 \mathrm{Fr}_{\mathrm{M}}\right)^{0,4}\left(1+1.3 / \mathrm{Fr}_{\mathrm{M}}^{0,6}\right)\left(T_{\mathrm{W}} / T_{\mathrm{f}}\right)^{-0,55} \tag{24}
\end{equation*}
$$

and refer it to (22), we arrive at the expression

$$
\begin{equation*}
A_{2}=\left(1.913 / \operatorname{Pr}^{0,4} \mathrm{Re}^{0,05}\right)\left(1+\pi^{2} / 0.5 \mathrm{Fr}_{\mathrm{M}}\right)^{0,4}\left(1+1.3 / \mathrm{Fr}_{\mathrm{M}}^{0,6}\right), \tag{25}
\end{equation*}
$$

which is in good agreement with (23) for Re $=10^{4}$ and $\mathrm{Fr}_{\mathrm{M}}=20-1000$. This fact indicates that the heat-transfer mechanisms are treated identically in Eqs. (15) and (21), but to within a constant factor. For example, if (15) is referred to (22), we deduce the relation

$$
\begin{equation*}
A_{3}=\left(1.52 / \operatorname{Pr}^{0,4} \operatorname{Re}^{0,05}\right)\left(1+\pi^{2} / 0.5 \mathrm{Fr}_{\mathrm{M}}\right)^{0,4}\left(1+1.3 / \mathrm{Fr}_{\mathrm{M}}^{0,6}\right) \tag{26}
\end{equation*}
$$

which well describes the experimental data obtained for bundles of 19 twisted tubes (Fig. 2) and gives smaller values than Eq. (23) for bundles with $N \geqslant 37$ tubes. This is consistent with the pattern represented in Fig. 1, where the data are also compared for bundles of 19 and $\mathrm{N} \geqslant 37$ tubes.

Also shown in Fig. 2 are the experimental data of [6] for seven-rod bundles at $\operatorname{Re}=10^{4}$. It is seen that the results of determining $N u$ for the central rod with a spiral wire-wrap spacer are in good agreement with (23), while the data on $N u$ for the peripheral rods are in good agreement with (26). This evinces the identical influence of the peripheral row of tubes on the heat-transfer coefficient in bundles of twisted tubes and spirally wire-wrapped rods at $\mathrm{Re} \approx 10^{4}$. The heat transfer for bundles with a large number of spirally ridged rods can be calculated according to Eqs. (4) and (21) only for $\operatorname{Re}<3 \cdot 10^{4}$. Thus, for $\operatorname{Re} \geqslant 3 \cdot 10^{4}$ a relative increase of Nu has been observed [6] in comparison with the function $\mathrm{Nu}=\mathrm{f}(\mathrm{Re})$ in the range of Reynolds numbers $\operatorname{Re}<3 \cdot 10^{4}$; this is clearly a peculiarity of the heat transfer in bundles of spirally wire-wrapped rods. The latter conjecture is clearly evident in Fig. 3, in which the experimental data of [6] for $R e=5 \cdot 10^{3}$ and $10^{5}$ are compared with

Eqs. (23), (25), and (26). Whereas the results of [6] practically coincide with the relations for bundles of twisted tubes (Fig. 3) for $\operatorname{Re}=5 \cdot 10^{3}$, as in the case $\operatorname{Re}=10^{4}$ (see Fig. 2), for $\operatorname{Re}=10^{5}$ the data of [6] for the central rod are much higher than Eqs. (23) and (25), while the data for the peripheral rods coincide with Eq. (23). On the other hand, Eq. (26) for $\operatorname{Re}=10^{5}$ is situated well below the curves for $\operatorname{Re}=5 \cdot 10^{3}$ and $10^{4}$. Clearly, it is not justified to extrapolate the data obtained for 19 -tube bundles at $\operatorname{Re} \leqslant 6 \cdot 10^{4}$ [3] and the data obtained for $N \geqslant 37$ at $\operatorname{Re} \leqslant 4 \cdot 10^{4}[1,2]$ to larger numbers Re, or the abrupt increase in the heat transfer for $\operatorname{Re}>3 \cdot 10^{4}$ in bundles of spirally ridged rods is a specific attribute of the flow in the investigated rod bundles.

It must be noted that the disparity of the experimental heat-transfer data for bundles with $N=19$ and $N \geqslant 37$ twisted tubes could be affected to some extent by the periodicity of the number Nu along the length of the bundles, including the stabilized flow section, as determined in [8]. This effect is associated with the three-dimensional aspect of the flow in a bundle of twisted tubes and the periodic variation of the flow conditions over the helical surface of the tubes along their length. The specific features of the heat transfer in the initial flow section in a bundle of twisted tubes with axisymmetrical entry of the heat-transfer agent have been discussed previously [8].

The foregoing analysis thus shows that the heat transfer of a twisted tube in a bundle depends on its position in the latter. The heat transfer is somewhat lower at the peripheral tubes next to the liner than for the central tubes. The reduction in the average heat transfer for the bundle under the influence of the peripheral tubes diminishes as the number of tubes in the bundle is increased. Consequently, Eqs. (21) or (4)-(7) are recommended for calculating the average heat transfer in a bundle with a large number of tubes (N $>37$ ). If the bundle contains a smaller number of tubes ( $N \leqslant 19$ ), Eq. (15) is recommended.

In the case of nonuniform heat input along the radius of a bundle of twisted tubes, the following relation $[2,9]$ can be used to calculate the heat transfer:

$$
\begin{equation*}
\alpha_{m}=\left(30.4 Z^{0,174} \operatorname{Pr}^{0,6}+14.65 Z^{0,09}-11.2\right)^{-1} \tag{27}
\end{equation*}
$$

where $\alpha_{m}$ is the dimensionless heat-transfer coefficient, which is defined as follows:

$$
\begin{gather*}
\alpha_{m}=\mathrm{Nu}_{\delta m} / \mathrm{Re}_{\delta m} \operatorname{Pr}_{m},  \tag{28}\\
Z=\mathrm{Re}_{\delta m}(1-2 \sqrt{\beta} / 0.39) / 0.39 \sqrt{\beta},  \tag{29}\\
\beta=0.045 Z^{-0,22 I}+4 \cdot 10^{-4} . \tag{30}
\end{gather*}
$$

The mean temperature over the thickness $\delta$ of the wall layer $T_{m}=0.5\left(T_{f}+T_{W}\right)$ is customarily taken as the reference in Eqs. (27)-(29), and the Reynolds number is determined from the velocity at the outer boundary of the wall layer.

The fluid friction in bundles of twisted tubes should not depend on the number of tubes when different procedures are used to process the experimental data. For example, the following relations have been recommended [1-3] for the calculation of the friction factor $\xi$ at $\mathrm{Fr}_{\mathrm{M}}>90$ :

$$
\begin{gather*}
\xi=\frac{0.3164}{\operatorname{Re}^{0,25}}\left(1+3.6 \mathrm{Fr}_{\mathrm{M}}^{-0,357}\right)  \tag{31}\\
\xi=0.266 / \operatorname{Re}_{8}^{0,25} \tag{32}
\end{gather*}
$$

If Eq. (31) is referred to the value $\xi_{t u}$ determined according to the Blasius equation for circular tubes, we arrive at Eq. (23), which is in good agreement for $\mathrm{Fr}_{\mathrm{M}}>90$ with the experimental data [1, 2, 10-12] for bundles of twisted tubes with $\mathrm{N}=19,37,127$ and $\operatorname{Re}=10^{4}$. The data of [3] on $\xi$ [Eq. (14)], referred to $\xi$ tu according to the Blasius equation, is farily consistent with (23) at $\operatorname{Re}=10^{4}$ for $\operatorname{FrM}=98$ and 235. However, for $\mathrm{Fr}_{\mathrm{M}}<90$, when an abrupt growth of the factor $\xi$ is observed according to the data of [11]:

$$
\begin{equation*}
\xi / \xi_{\mathrm{tu}}=\left(1+3.1 \cdot 10^{6} \mathrm{Fr}_{\mathrm{M}}^{-3,41}\right) \tag{33}
\end{equation*}
$$

the use of relation (14) yields far too low a value (Fig. 4). The data of [12] for $\mathrm{Fr}_{\mathrm{M}}=$ 56 agree with (33). The experimental data of [6] for a seven-rod bundle at $\mathrm{Fr}_{\mathrm{M}}=220$ and $\operatorname{Re}=10^{4}$ are in good agreement with (23), whereas at $\mathrm{Fr}_{\mathrm{M}}=860$ and 1900 they are much lower. This behavior of the function $\xi / \xi_{t u}=f\left(\mathrm{Fr}_{\mathrm{M}}\right)$ at $\operatorname{Re}=10^{4}$ for bundles of spirally wirewrapped rods is associated in all probability with the specific features of the flow in such bundles [6]. On the other hand, the data of [13] for bundles of spirally ridged rods are in good agreement with (23) and (33).

Thus, Eq. (31) can be recommended for calculating the friction factor in bundles of twisted tubes with $\operatorname{Fr}_{M} \geqslant 90$, and for $\mathrm{Fr}_{\mathrm{M}}<90$ the factor $\xi=3 \xi_{\text {tu }}$ can be used for engineering calculations with a certain margin of error.

## NOTATION

$\mathrm{Fr}_{\mathrm{M}}$, criterion characterizing the action of centrifugal forces on the flow; Nu, local Nusselt number; Re, Reynolds number; Pr, Prandtl number; $\mathrm{T}_{\mathrm{w}}$, wall temperature; $\mathrm{T}_{\mathrm{f}}$, flow temperature; $u$, velocity; $d e$, equivalent bundle diameter; $S$, pitch of twisted tube; d, maximum diameter of oval tube; $\rho$, density; $\mu$, dynamic viscosity coefficient; $\delta$, effective walllayer thickness; $x$, longitudinal coordinate with origin at the point of entry into the tube bundle; $z, y$, transverse coordinates; $q_{W}$, heat flux; $\lambda$, thermal conductivity; $l$, mixing length; $\tau$, tangential stresses; $\xi$, fluid friction factor; $\alpha$, heat-transfer coefficient; $Z$, special Reynolds number; $\alpha_{m}$, dimensionless heat-transfer coefficient; $N$, number of tubes in bundle. Indices: $\delta$, evaluated from the wall-layer thickness; w, wall; f, flow; T, turbulent; av, mass average (bulk value); m, evaluated at the mean temperature over the wall-1ayer thickness; $\tau$, transverse-tangential (transverse relative to axial component of tangential velocity or stress); tu, tube.

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## FLUID FRICTION OF TUBES FITTED WITH HELICAL HEAT-TRANSFER

## ENHANCERS

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The results of an analysis of experimental data on the fluid friction of tubes with heat-transfer enhancers are given. An empirical equation is proposed for generalization of the experimental data.

A number of papers have been published to date on the enhancement of heat transfer in tubes fitted with various types of enhancers. The latter are frequently in the form of helical wire-coil inserts, single-start and multistart helical rib roughening (ridging), and multistart helical corrugations [1-6]. Along with the investigation of the thermal characteristics of such tubes, these papers often present data on the fluid friction, which are needed in order to assess the engineering efficiency of heat exchangers constructed from the given tubes.

Inasmuch as heat transfer associated with turbulent fluid flow in tubes with helical enhancers is of the greatest practical interest, data on the fluid friction are generally given either in the form of a graph of $\xi_{h x}=f(R e)$ for $R e=3 \cdot 10^{4}-10^{5}$ [2] or in the form of an intricate dimensionless implicit function that generalizes only the specific data and is not suitable for practical engineering calculations [4].

In the present study, on the basis of an analysis of the available published experimental data, we attempt to derive a relation that is suitable for calculating fluid friction of tubes with helical enhancers and, if possible, will take into account the influence of the flow regime and the geometrical dimensions of the tube and enhancer, viz.: the tube diameter, the pitch of the helical insert or ridging, and the wire diameter or the ridge height.

We have selected for analysis the most complete friction data for 18 tubes with helical wire-coil inserts and helical ridging [1, 2]; these datawere obtained for roughly the same range of Reynolds numbers. The principal dimensions of the tubes are summarized in Table 1.

Figure 1 shows the dependence of the variation of the relative friction factor $\xi_{\text {hx }} / \xi_{s m}$, determined for the analyzed tubes at $\operatorname{Re}=2 \cdot 10^{4}$, on the parameter $d_{i n} / S$; the data are referred to the quantity $(h / S)^{m}$. The exponent $m$ characterizing the influence of the parameter $h / S$ has been evaluated [3] as $m=0.23 \mathrm{Re}^{0.07}$ and varied from 0.44 to 0.52 in the range of numbers $\operatorname{Re}=10^{4}-10^{5}$. A value $\mathrm{m}=0.5$ was taken in the first approximation. It is seen that the data in Fig. 1 are clustered with $\pm 17 \%$ scatter limits about an average line whose slope determines the degree of influence of the parameter $d_{i n} / S$ on the friction factor, i.e., $\xi_{\mathrm{hx}} \sim\left(\mathrm{d}_{\mathrm{in}} / \mathrm{S}\right){ }^{\mathrm{n}}$, where $\mathrm{n}=0.4$.
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